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All-versus-nothing nonlocality for two photons from different sources

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Abstract

A proof of all-versus-nothing nonlocality for two photons generated from two independent separated sources is presented. The reasoning makes use of entanglement of each photon with a vacuum state.

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The Bell theorem using an inequality [1] presents a statistical proof which refutes local hidden variable theories based on Einstein, Podolsky and Rosen's (EPR) local realism [2]. Strikingly, Greenberger, Horne and Zeilinger (GHZ) demonstrated the Bell theorem without inequalities or probabilities for three qubits and three observers [3, 4]. The quantum nonlocality can thus, in principle, be manifested in a single run of a certain measurement. This is known as the 'all-versus-nothing' proof of the Bell theorem [5]. Hardy [6] demonstrated nonlocality for nonmaximally entangled biparticle states, in which a fraction ($\lesssim 9\%$) of photon pairs shows a contradiction with local realism. Recently, Cabello [7, 8] showed that GHZ-type nonlocality can be proved for two observables who manipulate four two-level particles prepared in two pairs of singlet states, and this proof has been refined by Chen *et al* [9] in which two entangled photons and two degrees of freedom are involved.

In this paper we present a proof of all-versus-nothing nonlocality for two photons generated by two independent separated sources; so in this sense we say that two photons are unentangled. The reasoning makes use of entanglement of each photon with a vacuum state. Indeed, it has been experimentally demonstrated [10] that a single photon entangled with vacuum can violate local hidden variable models [11–13]. Figure 1 shows the schematic setup for generating two unentangled photons by two independent separated sources, then two photons would be manipulated by two observers, Alice and Bob, who are spacelike separated. When a single photon passes a balanced beam splitter (BS), it has equal probability amplitudes for reflection and transmission. In the particle number basis, such a state has the form (with a appropriate phase shifter) $\frac{1}{\sqrt{2}}(|1\rangle|0\rangle - |0\rangle|1\rangle)$, while this state is mathematically isomorphic to a two-photon Bell state encoded in horizontal and vertical polarization.

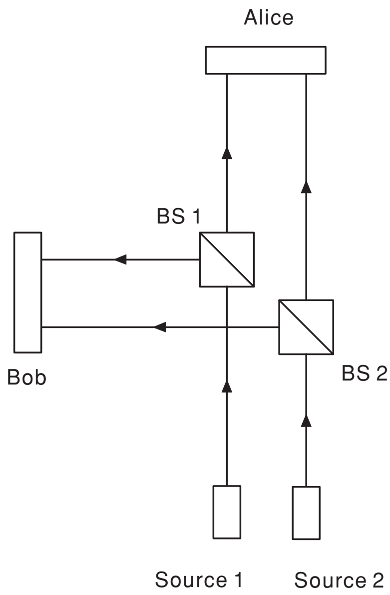

Figure 1. Schematic setup for generating state $|\Psi\rangle_{12}$.

Consider two photons prepared in the following state:

$$|\Psi\rangle_{12} = \frac{1}{2}\left(|1\rangle_A|0\rangle_B - |0\rangle_A|1\rangle_B\right)_1\left(|\bar{1}\rangle_A|\bar{0}\rangle_B - |\bar{0}\rangle_A|\bar{1}\rangle_B\right)_2, \tag{1}$$

where $|0\rangle_A$ ($|\bar{0}\rangle_A$) denotes a vacuum state entangled with first (second) photon $|1\rangle_A$ ($|\bar{1}\rangle_A$) possessed by Alice, and so on.

In the particle number basis one can define the following Pauli-type operators measured by Alice (Bob); for the first photon one has

$$\sigma_{xA(B)} = |1\rangle\langle 0| + |0\rangle\langle 1|, \tag{2}$$

$$\sigma_{zA(B)} = |1\rangle\langle 1| - |0\rangle\langle 0|, \tag{3}$$

and for second photon, we have

$$\bar{\sigma}_{xA(B)} = |\bar{1}\rangle\langle\bar{0}| + |\bar{0}\rangle\langle\bar{1}|, \tag{4}$$

$$\bar{\sigma}_{zA(B)} = |\bar{1}\rangle\langle\bar{1}| - |\bar{0}\rangle\langle\bar{0}|. \tag{5}$$

Note that, say, σ_{xA} measured by Alice equals a creation operator plus an annihilation operator; however, these local operations would not lead to violation of particle conservation as shown below.

By using the following notation, $z_i = \sigma_{zi}$, $x_i = \sigma_{xi}$, $\bar{z}_i = \bar{\sigma}_{zi}$, $\bar{x}_i = \bar{\sigma}_{xi}$ $(i = A, B)$, and using $(\cdot)$ to separate operators or operators' products that can be identified as EPR's local element of reality, one can easily check the following equations:

$$z_A \cdot z_B|\Psi\rangle_{12} = -|\Psi\rangle_{12}, \tag{6}$$

$$\bar{z}_A \cdot \bar{z}_B|\Psi\rangle_{12} = -|\Psi\rangle_{12}, \tag{7}$$

$$x_A \cdot x_B|\Psi\rangle_{12} = -|\Psi\rangle_{12}, \tag{8}$$

$$\bar{x}_A \cdot \bar{x}_B|\Psi\rangle_{12} = -|\Psi\rangle_{12}, \tag{9}$$

$$z_A\bar{z}_A \cdot z_B \cdot \bar{z}_B|\Psi\rangle_{12} = |\Psi\rangle_{12}, \tag{10}$$

$$x_A\bar{x}_A \cdot x_B \cdot \bar{x}_B|\Psi\rangle_{12} = |\Psi\rangle_{12}, \tag{11}$$

$$z_A \cdot \bar{x}_A \cdot z_B\bar{x}_B|\Psi\rangle_{12} = |\Psi\rangle_{12}, \tag{12}$$

$$x_A \cdot \bar{z}_A \cdot x_B\bar{z}_B|\Psi\rangle_{12} = |\Psi\rangle_{12}, \tag{13}$$

$$z_A\bar{z}_A \cdot x_A\bar{x}_A \cdot z_B\bar{x}_B \cdot x_B\bar{z}_B|\Psi\rangle_{12} = -|\Psi\rangle_{12}. \tag{14}$$

Equations (6)–(14) contain only local observables, i.e., ($z_A, \bar{z}_A, x_A, \bar{x}_A, z_A\bar{z}_A, x_A\bar{x}_A, z_A \cdot \bar{x}_A$, $x_A \cdot \bar{z}_A$ and $z_A\bar{z}_A \cdot x_A\bar{x}_A$) for Alice and ($z_B, \bar{z}_B, x_B, \bar{x}_B, z_B \cdot \bar{z}_B, x_B \cdot \bar{x}_B, z_B\bar{x}_B, x_B\bar{z}_B$ and $z_B\bar{x}_B \cdot x_B\bar{z}_B$) for Bob. In particular, equations (6)–(14) allow Alice (Bob) to assign values with certainty to Bob's local operators $z_B, \bar{z}_B, x_B, \bar{x}_B, z_B\bar{x}_B$ and $x_B\bar{z}_B$ (Alice's local operators $z_A, \bar{z}_A, x_A, \bar{x}_A, z_A\bar{z}_A$ and $x_A\bar{x}_A$) by measuring her (his) local observables without in any way disturbing Bob's (Alice's) photon. It is the idea of EPR's criterion of elements of reality to establish a local realistic interpretation of the quantum-mechanical results (6)–(14) by assuming that the individual value of any operator ($z_A, \bar{z}_A, x_A, \bar{x}_A, z_A\bar{z}_A$ and $x_A\bar{x}_A$) at Alice's side and ($z_B, \bar{z}_B, x_B, \bar{x}_B, z_B\bar{x}_B$ and $x_B\bar{z}_B$) at Bob's side is predetermined. These predetermined values are denoted by $v\,(z_i)$, $v\,(\bar{z}_i)$, $v\,(x_i)$, $v\,(\bar{x}_i)$, $v(z_A\bar{z}_A)$, $v(x_A\bar{x}_A)$, $v\,(z_B\bar{x}_B)$ and $v\,(x_B\bar{z}_B)$ with $v = \pm 1$. To be consistent with equations (6)–(14), local realistic theories thus predict

$$v(z_A)v(z_B) = -1, \tag{15}$$

$$v(\bar{z}_A)v(\bar{z}_B) = -1, \tag{16}$$

$$v\,(x_A)\,v\,(x_B) = -1 \tag{17}$$

$$v\,(\bar{x}_A)\,v\,(\bar{x}_B) = -1, \tag{18}$$

$$v\,(z_A\bar{z}_A)\,v\,(z_B)\,v\,(\bar{z}_B) = 1, \tag{19}$$

$$v\,(x_A\bar{x}_A)\,v\,(x_B)\,v\,(\bar{x}_B) = 1, \tag{20}$$

$$v\,(z_A)\,v\,(\bar{x}_A)\,v\,(z_B\bar{x}_B) = 1, \tag{21}$$

$$v\,(x_A)\,v\,(\bar{z}_A)\,v\,(x_B\bar{z}_B) = 1, \tag{22}$$

$$v\,(z_A\bar{z}_A)\,v\,(x_A\bar{x}_A)\,v\,(z_B\bar{x}_B)\,v\,(x_B\bar{z}_B) = -1. \tag{23}$$

But in fact, equations (15)–(23) are mutually inconsistent. Multiplying equations (15)–(22), one gets $v\,(z_A\bar{z}_A)\,v\,(x_A\bar{x}_A)\,v\,(z_B\bar{x}_B)\,v\,(x_B\bar{z}_B) = 1$ due to the fact that $v^2\,(z_i) = v^2\,(\bar{z}_i) = v^2\,(x_i) = v^2\,(\bar{x}_i) = 1$, and this is then in conflict with equation (23). Thus, the quantum-mechanical predictions (6)–(14) are incompatible with those imposed by local realistic theories. We therefore conclude that the predictions of quantum mechanics for a single copy of the state $|\Psi\rangle_{12}$ cannot be reproduced by any local realism of EPR. This completes the demonstration of an all-versus-nothing nonlocality for two photons generated from different sources.

Different from proof of [7–9] we use the photon numbers as the local observables, instead of photon polarization [7, 8] or photon polarization and spatial degrees of freedom [9]. Then a possible problem which may lead to argument on this demonstration is whether local measurements would violate particle conservation. Our argument is free from this problem since eigenequations (6)–(14) show that final states of two photons after measurements are invariant (up to a global phase).

Since two pairs of Bell states are employed in the above argument, a natural question is then: can we prove GHZ-type nonlocality for a Bell state? Unfortunately, the answer is negative, as Chen [14] has shown that the GHZ theorem cannot be extended to a Bell state.

Usually, entanglement of several particles is the necessary resource for demonstrating GHZ-type nonlocality. While our argument works for two unentangled photons in the sense that they have been prepared by two independent separated sources, and only one degree of freedom is employed. These features are essential for an experimental test of the GHZ-type theorem proposed here. So from the physical aspect, this proof is a further development of the Cabello-type proof [7–9] of quantum nonlocality without inequalities of two observers who possess some entangled two-level particles.

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